

# STBC Coded OFDM System Based on Turbo Product Codes

Yejun He, Guangxi Zhu, Feng Wang

Department of Electronics and Information Engineering  
 Huazhong University of Science and Technology, Wuhan, China  
 Tel: +86-027-87543236, Fax: +86-027-87548815  
 E-mail: heyeyun@126.com

**Abstract**—Space-Time Block Codes(STBCs) are designed to obtain the full diversity gain. However, STBCs are not designed to achieve an additional coding gain. Therefore, STBCs need to be concatenated with an outer code which provides a significant coding gain. Turbo Product Codes (TPCs) are a kind of high-efficient coding scheme with low latency decoding, which do not suffer from an error floor associated with turbo codes. This article considers the concatenation of TPCs with STBCs and sets up coded OFDM system to improve the reliability of the wireless communication systems. Finally, the paper presents simulating environment and results of experiment testing.

**Keywords**—turbo product codes; space-time block codes; OFDM; IMT2000 pedestrian channels

## I. INTRODUCTION

With the increasing demand for high-speed services in beyond third generation(B3G, including fourth generation, fifth generation et al) mobile communications systems, forward error correction(FEC) codes and signal processing techniques are receiving much attention.

It is known that orthogonal frequency division multiplexing (OFDM)<sup>[1]</sup>, multiple-input multiple-output (MIMO) and their combination are critical technologies in B3G mobile communications. Space-Time Block Code(STBC) is a implementation scheme of MIMO. Generally, there are three categories of MIMO techniques<sup>[2]</sup>. The first class includes delay diversity, STBC<sup>[3-4]</sup>, and space-time trellis code(STTC)<sup>[5]</sup>. The second class employes a layered approach to increase capacity, for example, V-BLAST system suggested by Foschini et al<sup>[6]</sup> where full spatial diversity is usually not achieved. The third type exploits the knowledge of channel at the transmitter. It decomposes the channel coefficient matrix using singular value decomposition(SVD)<sup>[7]</sup>.

The Space-Time Block Codes(STBCs) are designed to obtain the full diversity gain. However, STBCs are not designed to achieve an additional coding gain. Therefore, STBCs need to be concatenated with an outer code which provides a significant coding gain.

In mobile communication systems, if the first generation FEC is regarded as Reed-Solomon codes<sup>[8]</sup> and the second generation FEC is regarded as Turbo codes<sup>[9]</sup>, then the third

generation FEC is turbo product codes<sup>[10]</sup> and low density parity check(LDPC) codes<sup>[11]</sup>. TPCs and LDPC codes have been selected as FECs in satellite modulator/demodulator by Comtech company.

At present, domestic and overseas research literatures are either confined to the combination of turbo code and space time block code<sup>[12-14]</sup> as well as that of turbo code and BLAST<sup>[15]</sup>, or confined to the combination of turbo product code and space time block code<sup>[16-17]</sup> as well as that of turbo product code and OFDM<sup>[18]</sup>. TPC-STBC-OFDM system hasn't been reported over IMT2000 channels. In this paper, STBC proposed by Alamouti is employed.  $(64,57,4)^2$ ,  $(32,26,4)^2$ ,  $(16,11,4)^2$  TPCs are used as outer codes.

This paper is organized as follows. The System model is introduced in Section II. The encoding and decoding of TPC are analysed in Section III. In Section IV, A STBC coded OFDM system is described. Computer simulation results are presented to demonstrate the performance of the proposed STBC coded OFDM system based on turbo product codes over IMT-2000 pedestrian channels in Section V. Section VI contains the conclusions.

## II. STBC CODED OFDM SYSTEM BASED ON TURBO PRODUCT CODES

The STBC coded OFDM system based on turbo product codes is shown in Figure 1. An inverse FFT(IFFT) and a FFT is acted as a modulator and a demodulator, respectively. Figure 1 shows four paths with two transmit antennas and two receive antennas.

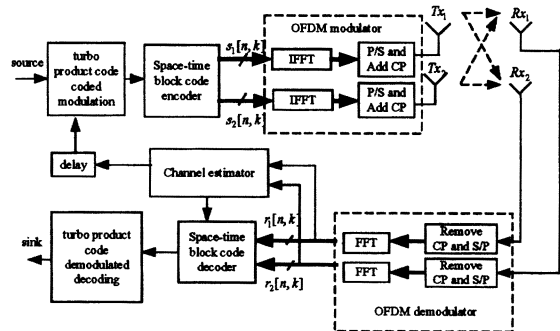


Figure 1. STBC coded OFDM system based on TPCs

Without loss of generality, the system with M transmit antennas and L receive antennas is considered. The received signal at each receive antenna is the superposition of M distorted transmitted signals, which can be expressed as

$$r_j[n, k] = \sum_{i=1}^M H_{ij}[n, k]s_i[n, k] + w_j[n, k] \quad (1)$$

where  $j=1, \dots, L, n=1, \dots, N, k=0, \dots, K-1$ .  $H_{ij}[n, k]$  denotes the channel frequency response for the  $k$ th tone at time  $n$ th, corresponding to the  $i$ th transmit and the  $j$ th receive antenna.  $s_i[n, k]$  is the input signal.

### III. THE ENCODING AND DECODING OF TURBO PRODUCT CODES(TPCs)

#### A. The encoding of TPCs

As an example of two-dimensional TPC  $c = c^1 \otimes c^2$ ,  $c^1 = (n_1, k_1, \delta_1)$ ,  $c^2 = (n_2, k_2, \delta_2)$ , where  $n_i, k_i$  and  $\delta_i$  ( $i=1,2$ ) are the codeword length, number of information bits, and minimum Hamming distance, respectively. The linear codes  $c^1$  and  $c^2$  are called constituent or component codes and the resultant  $(n_1 \times n_2, k_1 \times k_2, \delta_1 \times \delta_2)$  TPC has a code rate of  $R = R_1 \times R_2 = (k_1 \times k_2) / (n_1 \times n_2)$ . Two-dimensional TPC can be obtained as follows:

- 1) placing a  $k_1 \times k_2$  array of information bits as shown in Figure 2.
- 2)  $k_2$  rows of  $k_1$  information bits are encoded by row encoder  $c^1$  to get a matrix of  $k_2$  rows and  $n_1$  columns.
- 3)  $n_1$  columns of the matrix are encoded by column encoder  $c^2$  to get a matrix of  $n_2$  rows and  $n_1$  columns.

The above step 2) and 3) cannot be performed at the same time, therefore the encoding scheme is called as serial concatenation. To decrease implementation complexity, the rows and columns are encoded by the same constituent code.

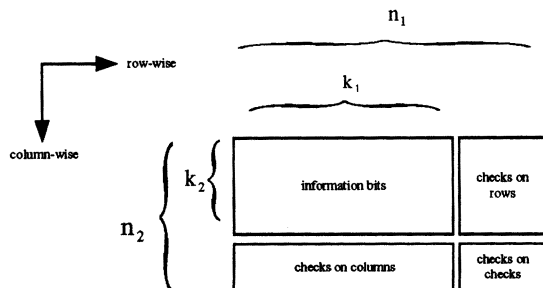


Figure 2. The encoding block of two-dimensional TPCs

#### B. The decoding of TPCs

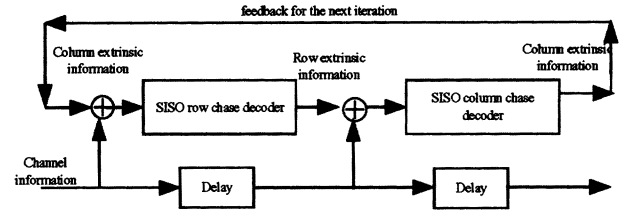


Figure 3. The serial iterative decoder of two-dimensional TPC

The decoding algorithm is based on soft-decision decoding of the constituent codes and high error-correcting performance is achieved by decoding row-wise and column-wise of TPC array repeatedly. As shown in Figure 3,a full-iteration of a TPC decoder performs row and column decoding in a serial fashion. The whole product codeword needs to be decoded row-wise or column-wise for N times before the next half-iteration can begin, where N equals the column number and the row number of the product code array.

#### C. Bound on performance of two-dimensional TPCs

The maximum likelihood asymptote (MLA) for a two-dimensional TPCs can be written as<sup>[19]</sup>

$$P_{MLA} = \frac{\delta_1 \delta_2}{n_1 n_2} A_{\delta_1}(c^1) A_{\delta_2}(c^2) Q \left( \sqrt{\frac{2E_b}{N_0} R_1 R_2 \delta_1 \delta_2} \right) \quad (2)$$

where  $E_b$  is the information bit energy,  $R_1$  and  $R_2$  are code rate of  $c^1 = (n_1, k_1, \delta_1)$  and  $c^2 = (n_2, k_2, \delta_2)$ , respectively.

This asymptote is obtained by considering only those code words that have minimum Hamming weight. Using the weight enumerator for extended Hamming code<sup>[20]</sup>, we can show that for an extended (64,57,4), (32,26,4), (16,11,4) Hamming codes,  $A_4(64,57,4)=10416$ ,  $A_4(32,26,4)=1240$ , and  $A_4(16,11,4)=140$ . Here, the row and column component codes of the TPC are the same. Therefore, we get the MLA for two-dimensional TPCs as described in Figure 4.

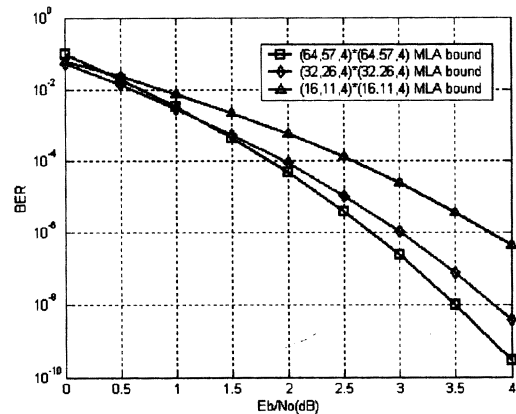


Figure 4. Maximum likelihood asymptote for (16,11,4)<sup>2</sup>, (32,26,4)<sup>2</sup> and (64,57,4)<sup>2</sup> TPCs

IV. THE SIMULATION IMPLEMENTATION OF STBC CODED OFDM SYSTEM WITH TWO TRANSMIT ANTENNAS AND TWO RECEIVE ANTENNAS

A STBC coded OFDM system with two transmit antennas and two receive antennas is proposed in Figure 5. An OFDM symbol is denoted a complex vector  $s(n)$ , where  $n$  is the discrete time index for  $t = nT_f$ , with  $T_f$  being the OFDM symbol interval. There are  $N_s$  complex subsymbol in  $s(n)$  denoted as  $s[n, k], k=1, 2, \dots, N_s$ , where  $N_s$  is the total number of subchannels in OFDM systems. For subchannel  $k$ , assume two complex vectors  $S_1[n, k], S_2[n, k]$  after STBC encoding enter two OFDM modulators, respectively. In the next time, two complex vectors are denoted as  $S_1[n+1, k], S_2[n+1, k]$ , the STBC encoder works as follows:

$$\begin{aligned} S_1[n, k] &= s[n, k] \\ S_2[n, k] &= s[n+1, k] \\ S_1[n+1, k] &= -s^*[n+1, k] \\ S_2[n+1, k] &= s^*[n, k] \end{aligned} \quad (3)$$

where, \* denotes complex conjugate.

As a result, the received subsymbol at subchannel  $k$  from receive antenna  $j=1, 2$  after FFT demodulation are

$$\begin{aligned} R_j[n, k] &= H_{1j}(k)S_1[n, k] + H_{2j}(k)S_2[n, k] + W_j[n, k] \\ &= H_{1j}(k)s[n, k] + H_{2j}(k)s[n+1, k] + W_j[n, k] \end{aligned} \quad (4)$$

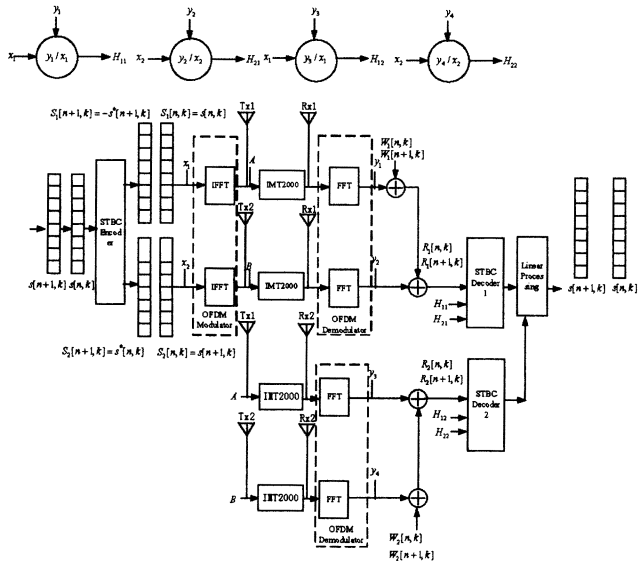


Figure 5. Simulation implementation block diagram of STBC coded OFDM system

$$\begin{aligned} R_j[n+1, k] &= H_{1j}(k)S_1[n+1, k] + H_{2j}(k)S_2[n+1, k] + W_j[n+1, k] \\ &= -H_{1j}(k)s^*[n+1, k] + H_{2j}(k)s^*[n, k] + W_j[n+1, k] \end{aligned} \quad (5)$$

and the combiner for receive antenna  $j$  is

$$\begin{aligned} \mathcal{S}_j[n, k] &= H_{1j}^*(k)R_j[n, k] + H_{2j}(k)R_j^*[n+1, k] \\ \mathcal{S}_j[n+1, k] &= H_{2j}^*(k)R_j[n, k] - H_{1j}(k)R_j^*[n+1, k] \end{aligned} \quad (6)$$

In STBC decoder, the combined signals are then sent to the maximum likelihood detector to perform signal detection. Finally, using the linear processing unit after two STBC decoder, we get  $s[n, k], s[n+1, k]$ .

V. NUMERICAL SIMULATION

In terms of the proposed scheme in this paper, Each path is modeled as IMT-2000 pedestrian channels, which include pedestrian a, pedestrian b, Alamouti STBC is employed. The channel estimator is assumed to be ideal. The simulation parameters are shown in Table 1. The Simulations were performed on a Signal Processing Worksystem (SPW) software. The simulation results are shown in Figure 6-7. Signal to noise ratio is defined as  $E_s/N_0 = 2E_p/N_0$ , where,  $E_p$  denotes signal power per transmit antenna,  $N_0$  denotes noise power.

TABLE I. SIMULATION PARAMETERS

Parameters	Specification
FFT sizes	1024
Modulation	BPSK
TPCs Structure	$(16, 11, 4)^2$ , $(32, 26, 4)^2$ , $(64, 57, 4)^2$
Cyclic prefix interval	64
Guard interval	128
Carrier Frequency( $f_c$ )	2GHz
Sampling Frequency	100MHz
The signal power per transmit antenna	$P_s = 1$
Channel Model	IMT2000 pedestrian a(5km/h) IMT2000 pedestrian b(5km/h)
Iterative number of TPCs	16
Default Doppler Frequency	$(\text{speed} * f_c) / (2.997 \times 10^8 \times 3.6)$
Number of antenna	2 Transmit antennas and 2 receive antennas

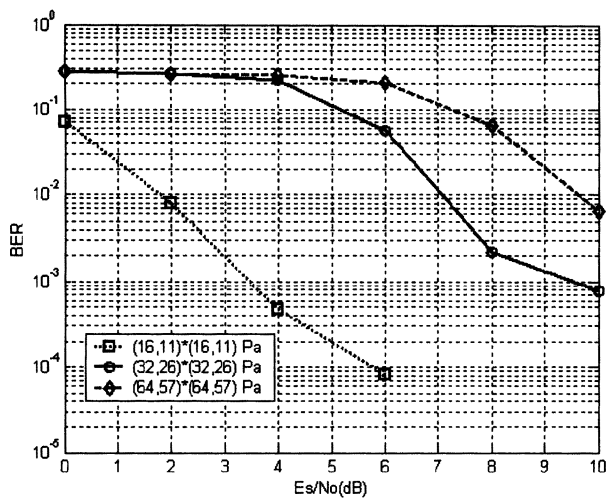


Figure 6. Performance of TPC-STBC- OFDM system over IMT2000-Pa

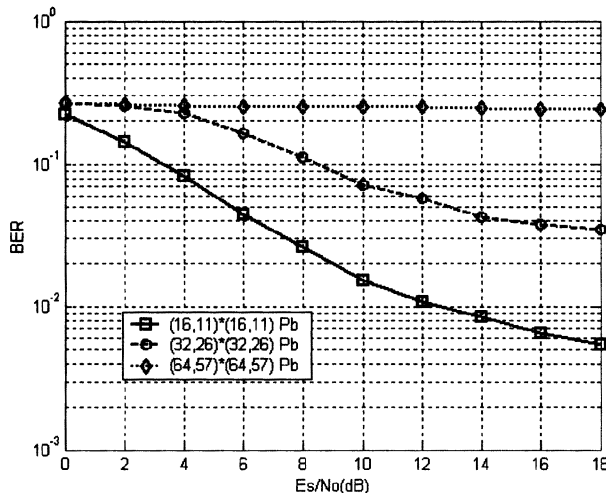


Figure 7. Performance of TPC-STBC- OFDM system over IMT2000-Pb

## VI. CONCLUSIONS

In this paper we presented a STBC coded OFDM system based turbo product codes (TPCs). This system has been tested on several TPCs based on extended Hamming codes. Simulation results show that:

1) System performance for low code rate TPCs is better than that for high code rate TPCs in the same channel. There are an identical variety trend for each channel, for example, the BER curve of  $(16,11,4)^2$  TPC is under that of  $(32,26,4)^2$  TPC.

2) Using the identical TPC, system performance for IMT-2000 pedestrian A (Pa) channel is better than that for IMT-2000 pedestrian B (Pb). The result conforms to the channel properties set by ETSI to test UMTS (Universal

Mobile Telecommunication System) as third generation standardization.

3) TPC decoder influences on the proposed system performance.

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